ESTIMATION OF STRESS-STRENGTH MODEL FOR GENERALIZED INVERTED EXPONENTIAL DISTRIBUTION USING RANKED SET SAMPLING

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ABSTRACT

In this paper, the estimation of R=P(Y < X), when X and Y are two generalized inverted exponential random variables with different parameters is considered. This problem arises naturally in the area of reliability for a system with strength X and stress Y. The estimation is made using simple random sampling (SRS) and ranked set sampling (RSS) approaches. The maximum likelihood estimator (MLE) of R is derived using both approaches. Assuming that the common scale parameter is known, MLEs of R are obtained. Monte Carlo simulations are performed to compare the estimators obtained using both approaches. . The properties of these estimators are investigated and compared with known estimators based on simple random sample (SRS) data. The comparison is based on biases, mean squared errors (MSEs) and the efficiency of the estimators of R based on RSS with respect to those based on SRS. The estimators based on RSS is found to dominate those based on SRS.

KEYWORDS: *generalized exponential distribution; reliability; stress-strength; ranked set sampling, simple random sampling; maximum likelihood estimators.*

I. INTRODUCTION

The estimation of reliability is a very common problem in statistical literature. The most widely approach applied for reliability estimation is the well-known stress-strength model. This model is used in many applications of physics and engineering such as strength failure and the system collapse. In the stress-strength modeling, *R*=P(Y < X) is a measure of component reliability when it is subjected to random stress Y and has strength X. In this context, *R* can be considered as a measure of system performance and it is naturally arise in electrical and electronic systems. Another interpretation can be that, the reliability of the system is the probability that the system is strong enough to overcome the stress imposed on it. It may be mentioned that *R* is of greater interest than just reliability since it provides a general measure of the difference between two populations and has applications in many areas. For example, if X is the response for a control group, and *Y* refers to a treatment group, *R* is a measure of the effect of the treatment. In addition, it may be mentioned that *R* equals the area under the receiver operating characteristic (ROC) curve for diagnostic test or biomarkers with continuous outcome, see; Bamber, [1]. The ROC curve is widely used, in biological, medical and health service research, to evaluate the ability of diagnostic tests or biomarkers and to distinguish between two groups of subjects, usually non-diseased and diseased subjects. For complete review and more applications of *R;* see [2-14].

Ranked set sampling (RSS) is a sampling protocol that can often be used to improve the cost and efficiency for experiments [15]. It is often used when a ranking of the sampling units can be obtained cheaply without having to actually measure the characteristics of interest, which may be time consuming or costly [16,17]. Such a technique is well received and widely applicable in environmental applications, reliability and quality control experiments [18-20]. A modification of ranked set sampling (RSS) called moving extremes ranked set sampling (MERSS) was considered for

the estimation of the scale parameter of scale distributions [21] and an improved RSS estimator for the population mean was obtained [22]. On the other hand, Ozturk has developed two sampling designs to create artificially stratified samples using RSS [23]

Recently, many authors have been interested in estimating *R* using RSS. For example, Sengupta and Mukhuti [24], considered an unbiased estimation of *R* using RSS for exponential populations. Muttlak and co-authors [25], proposed three estimators of *R* using RSS when X and Y independent one-parameter exponential populations. In a RSS procedure, *m* independent sets of SRS each of size *m* are drawn from the distribution under consideration. these samples are ranked by some auxiliary criterion that does not require actual measurements and only the i^{th} smallest observation is quantified from the i^{th} set, $i = 1,2,\ldots,m$. This completes a cycle of the sampling. Then, the cycle is repeated *k* times to obtain a ranked set sample of size $n = m\,k$.

In this article, we consider the estimation of *R*, when X and Y are independent but not identically distributed generalized inverted exponential distribution (GIED) variables. Abouammoh and Alshingiti [26], introduced a shape parameter in the inverted exponential distribution (IED) to obtain the generalized inverted exponential distribution (GIED). They derived many distributional properties and reliability characteristics of GIED. Assuming it to be a good lifetime model, they obtained maximum likelihood estimators, least square estimators and confidence intervals of the two parameters involved. The GIED has the following cumulative distribution function (*cdf*) and probability density function (*pdf*) for X > 0:

$$F(x) = 1-[1-\exp(-\lambda/x)]^{\alpha}, \qquad x>0\ , \tag{1}$$

with

$$f(x) = \alpha\frac{\lambda}{x^2}\exp(-\lambda/x)[1-\exp(-\lambda/x)]^{\alpha-1},\ x>0\ , \tag{2}$$

where $\lambda > 0$ is the scale parameter and $\alpha > 0$ is the shape parameter.

The rest of paper is organized as follows: the derivation of the stress-strength (*R* = P(Y<X)) model for the generalized inverted exponential distribution is discussed. In sections 3 and 4, the MLEs of *R* using SRS and RSS approaches are derived.. In section 5 simulation studies and comparison between estimators obtained using both approaches are discussed. Finally, the paper is concluded in section 6.

II. THE STRESS STRENGTH MODEL

Let X and Y be two independent GIED random variables with parameters (λ,α) and (λ,β) respectively. Therefore, the reliability of the system is given by

$$\begin{aligned} R = P(Y<X) &= \int_0^1 f_X(x)P(Y<X\setminus X=x)\,dx \\ &= \int_0^\infty \alpha\frac{\lambda}{x^2}\exp(-\lambda/x)[1-\exp(-\lambda/x)]^{\alpha-1}(1-[1-\exp(-\lambda/x)]^{\beta})\,dx\,, \\ &= 1-\frac{\alpha}{\alpha+\beta} = \frac{\beta}{\alpha+\beta}\,. \end{aligned} \tag{3}$$

III. MAXIMUM LIKELIHOOD ESTIMATION OF R USING SRS

Let $(X_1, X_2,\ldots,X_n)$ and $(Y_1,Y_2,\ldots,Y_m)$ be two independent random samples from GIED(λ,α) and GIED(λ,β) respectively. The likelihood function of λ, α and β for the observed samples is

$$\begin{aligned} L_S(data;\lambda,\alpha,\beta) = {}&\lambda^n\alpha^n[\prod_{i=1}^{n}x_i^2]^{-1}\exp(-\sum_{i=1}^{n}\lambda/x_i)\prod_{i=1}^{n}[1-\exp(-\lambda/x_i)]^{\alpha-1} \\ &\times\ \lambda^m\beta^m[\prod_{j=1}^{m}y_j^2]^{-1}\exp(-\sum_{j=1}^{m}\lambda/y_j)\prod_{j=1}^{m}[1-\exp(-\lambda/y_j)]^{\beta-1} \end{aligned} \tag{4}$$

Therefore, the log-likelihood function of λ, α and β will be

$$\log L_S = (m+n)\log\lambda + n\log\alpha + m\log\beta - 2\sum_{i=1}^{n}\log x_i - \sum_{i=1}^{n}\frac{\lambda}{x_i} + (\alpha-1)\sum_{i=1}^{n}\log[1-\exp(-\lambda/x_i)]$$
$$-2\sum_{j=1}^{m}\log y_j - \sum_{j=1}^{m}\frac{\lambda}{y_j} + (\beta-1)\sum_{j=1}^{m}\log[1-\exp(-\lambda/y_j)]. \quad (5)$$

The estimators $\hat{\lambda}_s$, $\hat{\alpha}_s$ and $\hat{\beta}_s$ of the parameters λ, α and β respectively can be then obtained as the solution of the likelihood equations

$$\frac{(n+m)}{\lambda} - \sum_{i=1}^{n}\frac{1}{x_i} - \sum_{j=1}^{m}\frac{1}{y_j} + (\alpha-1)\sum_{j=1}^{m}\frac{\exp(-\lambda/x_i)}{x_i\,[1-\exp(-\lambda/x_i)]} + (\beta-1)\sum_{j=1}^{m}\frac{\exp(-\lambda/y_j)}{y_j\,[1-\exp(-\lambda/y_j)]} = 0 \quad (6)$$

$$\frac{n}{\alpha} + \sum_{i=1}^{n}\log[1-\exp(-\lambda/x_i)] = 0 \quad (7)$$

$$\frac{m}{\beta} + \sum_{j=1}^{m}\log[1-\exp(-\lambda/y_j)] = 0 \quad (8)$$

Once the nonlinear equations (6-8) have been solved with respect to λ, α and β, the MLEs $\hat{\alpha}_s$ and $\hat{\beta}_s$ will be given as

$$\hat{\alpha}_s = -\frac{n}{\sum_{i=1}^{n}\log[1-\exp(-\hat{\lambda}_s/x_i)]}, \quad (9)$$

and

$$\hat{\beta}_s = -\frac{m}{\sum_{j=1}^{m}\log[1-\exp(-\hat{\lambda}_s/y_j)]}, \quad (10)$$

where $\hat{\lambda}_s$ is the solution of the nonlinear equation,

$$\frac{(n+m)}{\hat{\lambda}_s} - \sum_{i=1}^{n}\frac{1}{x_i} - \sum_{j=1}^{m}\frac{1}{y_j} - \left(\frac{n}{\sum_{i=1}^{n}\log[1-\exp(-\hat{\lambda}_s/x_i)]}+1\right)\sum_{j=1}^{m}\frac{\exp(-\hat{\lambda}_s/x_i)}{x_i\,[1-\exp(-\hat{\lambda}_s/x_i)]}$$
$$-\left(\frac{m}{\sum_{j=1}^{m}\log[1-\exp(-\hat{\lambda}_s/y_j)]}+1\right)\sum_{j=1}^{m}\frac{\exp(-\hat{\lambda}_s/y_j}{y_j\,[1-\exp(-\hat{\lambda}_s/y_j)]} = 0. \quad (11)$$

By the invariance property of the MLEs, the MLE of R becomes

$$\hat{R}_s = \frac{\hat{\beta}_s}{\hat{\alpha}_s+\hat{\beta}_s} = \frac{[m\sum_{i=1}^{n}\log(1-\exp(-\hat{\lambda}_s/x_i)]}{[m\sum_{i=1}^{n}\log(1-\exp(-\hat{\lambda}_s/x_i)+n\sum_{j=1}^{m}\log[1-\exp(-\hat{\lambda}_s/y_j)]}. \quad (12)$$

Assuming the scale parameter is known, the MLEs $\hat{\alpha}_{s,\lambda}$, $\hat{\beta}_{s,\lambda}$ and $\hat{R}_{s,\lambda}$ of the parameters α, β and R are respectively

$$\hat{\alpha}_{s,\lambda} = \frac{-n}{\sum_{i=1}^{n}\log[1-\exp(-\lambda/x_i)]}, \quad (13)$$

$$\hat{\beta}_{s,\lambda} = \frac{-m}{\sum_{j=1}^{m}\log[1-\exp(-\lambda/y_j)]}, \quad (14)$$

and

$$\hat{R}_{s,\lambda} = \frac{[m\sum_{i=1}^{n}\log(1-\exp(-\lambda/x_i)]}{[m\sum_{i=1}^{n}\log(1-\exp(-\lambda/x_i)+n\sum_{j=1}^{m}\log[1-\exp(-\lambda/y_j)]}. \quad (15)$$

IV. MAXIMUM LIKELIHOOD ESTIMATION OF R USING RSS

Assume that $X_{(i:m_x)j}, i=1,\ldots,m_x,\ j=1,\ldots,r_x$ is a ranked set samples with sample size $n_x = m_x r_x$ drawn from GIED with shape and scale parameters α and λ respectively. Let $Y_{(k:m_y)l}, k=1,\ldots,m_y, l=1,\ldots,r_y$ be an independent ranked set sample with sample size $n_y = m_y r_y$ drawn from GIED with shape and scale parameters β and λ respectively. m_x and r_x are the set size and the number of cycles of the RSS sample drawn from GIED(λ,α), while m_y and r_y are the set size and the number of cycles of the RSS drawn from GIED(λ,β). For simplification purposes, $X_{(i:m)j}$ and $Y_{(k:m)l}$ will be denoted as X_{ij} and Y_{kl} respectively. The *pdf* of the random variables X_{ij} and Y_{kl} are given by

$$g_1(x_{ij}) = \frac{m_x!}{(i-1)!(m_x-i)}\alpha\frac{\lambda}{x_{ij}^2}\exp(-\lambda/x_{ij})[1-\exp(-\lambda/x_{ij}]^{\alpha(m_x-i+1)-1}(1-[1-\exp(-\lambda/x_{ij}]^{\alpha})^{m_x-i}, \tag{16}$$

$$g_2(y_{kl}) = \frac{m_y!}{(k-1)!(m_y-k)}\beta\frac{\lambda}{y_{kl}^2}\exp(-\lambda/y_{kl})[1-\exp(-\lambda/y_{kl})]^{\beta(m_y-k+1)-1}(1-[1-\exp(-\lambda/y_{kl})]^{\beta})^{m_y-k}, \tag{17}$$

where $\lambda,\alpha,\beta>0$, $X_{ij}>0$, $Y_{kl}>0$, $j=1,\ldots,r_x$ and $l=1,\ldots,r_y$. The likelihood function of λ, α and β for the observed samples is given by

$$L_r(data;\alpha,\lambda) = K\prod_{j=1}^{r_x}\prod_{i=1}^{m_x}\exp(-\lambda/x_{ij})[1-\exp(-\lambda/x_{ij}]^{\alpha(m_x-i+1)-1}(1-[1-\exp(-\lambda/x_{ij}]^{\alpha})^{m_x-i}$$

$$\times\prod_{l=1}^{r_y}\prod_{k=1}^{m_y}\beta\frac{\lambda}{y_{kl}^2}\exp(-\lambda/y_{kl})[1-\exp(-\lambda/y_{kl}]^{\beta(m_y-k+1)-1}(1-[1-\exp(-\lambda/y_{kl})]^{\beta})^{m_y-k} \tag{18}$$

Therefore, the log-likelihood function of λ, α and β will be

$$Log\,L_r = K^* + m_x r_x(\log\alpha+\log\lambda) + m_y r_y(\log\beta+\log\lambda) - \sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\lambda/x_{ij} - \sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\lambda/y_{kl}$$

$$+(\alpha(m_x-i+1)-1)\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\log[1-\exp(-\lambda/x_{ij})] + (\beta(m_y-k+1)-1)\sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\log[1-\exp(-\lambda/y_{kl})]$$

$$+(m_x-i)\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\log(1-[1-\exp(-\lambda/x_{ij})]^{\alpha}) + (m_y-k)\sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\log(1-[1-\exp(-\lambda/y_{kl})]^{\beta}) \tag{19}$$

where K^* is a constant. This implies that

$$\frac{m_x r_x}{\alpha} + (m_x-i+1)\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\log[1-\exp(-\lambda/x_{ij})] + (m_x-i)\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\frac{[1-\exp(-\lambda/x_{ij})]^{\alpha}\log[1-\exp(-\lambda/x_{ij})]}{1-[1-\exp(-\lambda/x_{ij})]^{\alpha}} = 0, \tag{20}$$

$$\frac{m_y r_y}{\beta} + (m_y-k+1)\sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\log[1-\exp(-\lambda/y_{kl})] + (m_y-k)\sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\frac{[1-\exp(-\lambda/y_{kl})]^{\beta}\log[1-\exp(-\lambda/y_{kl})]}{1-[1-\exp(-\lambda/y_{kl})]^{\beta}} = 0, \tag{21}$$

and

$$\frac{m_x r_x + m_y r_y}{\lambda} - \sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\frac{1}{x_{ij}} - \sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\frac{1}{y_{kl}} + (\alpha(m_x-i+1)-1)\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\frac{\exp(-\lambda/x_{ij})}{x_{ij}[1-\exp(-\lambda/x_{ij}]}$$

$$+(\beta(m_y-k+1)-1)\sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\frac{\exp(-\lambda/y_{kl})}{y_{kl}[1-\exp(-\lambda/y_{kl})]} + \alpha(m_x-i)\sum_{j=1}^{r}\sum_{i=1}^{m}\frac{\exp(-\lambda/x_{ij})[1-\exp(-\lambda/x_{ij})]^{\alpha-1}}{x_{ij}(1-[1-\exp(-\lambda/x_{ij})]^{\alpha})}$$

$$+\beta(m_y-k)\sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\frac{\exp(-\lambda/y_{kl})[1-\exp(-\lambda/y_{kl})]^{\beta-1}}{y_{kl}(1-[1-\exp(-\lambda/y_{kl})]^{\beta})} = 0. \tag{22}$$

To derive the MLEs $\hat{\lambda}_r$ $\hat{\alpha}_r$ and $\hat{\beta}_r$ of the parameters λ, α and β, we have to solve the nonlinear equations (20-22) with respect to λ, α and β. Therefore, the MLE of *R* is given by

$$\hat{R}_r = \frac{\hat{\beta}_r}{\hat{\alpha}_r + \hat{\beta}_r} \tag{23}$$

Once again, assuming that the scale parameter λ is known, the MLEs $\hat{\alpha}_{r,\lambda}$ and $\hat{\beta}_{r,\lambda}$ of the parameters α and β are the solution of the likelihood equations

$$\frac{m_x r_x}{\hat{\alpha}_{r,\lambda}} + (m_x - i + 1)\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\log[1-\exp(-\lambda / X_{ij})] + (m_x - i)\sum_{j=1}^{r_x}\sum_{i=1}^{m_x}\frac{[1-\exp(-\lambda / x_{ij})]^{\hat{\alpha}_{r,\lambda}}\log[1-\exp(-\lambda / x_{ij})]}{1-[1-\exp(-\lambda / x_{ij})]^{\hat{\alpha}_{r,\lambda}}} = 0 \quad (24)$$

$$\frac{m_y r_y}{\hat{\beta}_{r,\lambda}} + (m_y - k + 1)\sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\log[1-\exp(-\lambda / y_{kl})] + (m_y - k)\sum_{l=1}^{r_y}\sum_{k=1}^{m_y}\frac{[1-\exp(-\lambda / y_{kl})]^{\hat{\beta}_{r,\lambda}}\log[1-\exp(-\lambda / y_{kl})]}{1-[1-\exp(-\lambda / y_{kl})]^{\hat{\beta}_{r,\lambda}}} = 0 \quad (25)$$

Thus, the MLE of *R* is given by

$$\hat{R}_{r,\lambda} = \frac{\hat{\beta}_{r,\lambda}}{\hat{\alpha}_{r,\lambda} + \hat{\beta}_{r,\lambda}} \quad (26)$$

It is clear that the MLEs of λ and α denoted by $\hat{\lambda}_r$ and $\hat{\alpha}_r$ cannot be obtained in closed form. Thus, we will use simulation to study the properties of these estimators and the properties of the estimators of *R*.

V. SIMULATION STUDY

This section presents a numerical comparison between the MLEs of *R*=P(Y < X) using both SRS and RSS approaches. The estimation is made through biases and mean squared errors (MSE) of the estimators $\hat{R}_{s,\lambda}$ and $\hat{R}_{r,\lambda}$ and the efficiency of the estimator $\hat{R}_{r,\lambda}$ with respect the estimator $\hat{R}_{s,\lambda}$ where the efficiency of a parameter θ_2 with respect to the parameter θ_1 is given by

$$eff(\hat{\theta}_1, \hat{\theta}_2) = \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)} \quad (27)$$

The simulation studies are made using MATHEMATICA v.8 for several combinations of the parameters n, m, m_x, r_x, m_y, r_y and *R* and for $r_x = r_y = 5$. The random samples of GIED(α,λ) and GIED(β,λ) are generated using the forms

$$x = -\frac{\lambda}{\log[1-u^{1/\alpha}]}, \qquad x \geq 0, \quad \lambda, \alpha > 0 \quad (28)$$

and

$$y = -\frac{\lambda}{\log[1-v^{1/\beta}]}, \qquad x \geq 0, \quad \lambda, \beta > 0 \quad (29)$$

where $0 < u < 1$ and $0 < v < 1$ are uniform random variables. Different simulations are based on 1000 replications. The results are shown in Tables 1-3.

Based on Tables 1-3, one can see that the biases and MSEs of both $\hat{R}_{s,\lambda}$ and $\hat{R}_{r,\lambda}$ decreases as the sample size increases which indicates the asymptotic consistency property for the MLEs. It can be observed that as the reliability parameter *R* increases, the biases and MSEs for both $\hat{R}_{s,\lambda}$ and $\hat{R}_{r,\lambda}$ decreases. It is also observed that for small values of the scale parameter λ, $0 < \lambda \leq 1$(Tables 1 and 2), estimation of *R* using both SRS and RSS approaches is better than estimation when $\lambda > 1$ (Table3). Comparing the two approaches, the biases and MSEs of $\hat{R}_{r,\lambda}$ is always better than those of $\hat{R}_{s,\lambda}$ which can be noted from the efficiency of $\hat{R}_{r,\lambda}$ with respect to $\hat{R}_{s,\lambda}$. Finally, the efficiency of $\hat{R}_{r,\lambda}$ is greater than one and increases as the sample sizes increase.

Table 1: MLE estimation of *R* when different values of the parameters *α* and *β* and when the scale parameter is known ($\lambda = 0.5$) and $r_x = r_y = 5$.

R	*(n,m)*	m_x, m_y	$Bias(\hat{R}_{s,\lambda})$	$Bias(\hat{R}_{r,\lambda})$	$MSE(\hat{R}_{s,\lambda})$	$MSE(\hat{R}_{r,\lambda})$	$eff(\hat{R}_{r,\lambda})$
0.250	(10,10)	(2,2)	0.0699	0.0546	1.6481	1.4317	1.1511
	(10,15)	(2,3)	0.0635	0.0496	1.2180	0.9648	1.2626
	(10,25)	(2,5)	0.0585	0.0477	0.9625	0.7376	1.3056

	(15,25)	(3,5)	0.0551	0.0441	0.7336	0.5507	1.3319
	(25,25)	(5,5)	0.0512	0.0398	0.6122	0.4154	1.4738
0.333	(10,10)	(2,2)	0.0543	0.0354	1.5443	1.3605	1.1353
	(10,15)	(2,3)	0.0429	0.0291	1.11885	0.9167	1.2204
	(10,25)	(2,5)	0.0409	0.0267	0.8958	0.7006	1.2786
	(15,25)	(3,5)	0.0389	0.0249	0.6817	0.5235	1.3024
	(25,25)	(5,5)	0.0325	0.0221	0.5704	0.3947	1.4453
0.500	(10,10)	(2,2)	0.0423	0.0305	1.4764	1.2079	1.2223
	(10,15)	(2,3)	0.0358	0.0258	1.0912	0.8137	1.3405
	(10,25)	(2,5)	0.0299	0.0225	0.8623	0.6219	1.3864
	(15,25)	(3,5)	0.0271	0.0203	0.6572	0.4646	1.4143
	(25,25)	(5,5)	0.0243	0.0196	0.5484	0.3504	1.5650
0.667	(10,10)	(2,2)	0.0391	0.0254	1.3985	1.2067	1.1589
	(10,15)	(2,3)	0.0308	0.0209	1.0023	0.7891	1.2702
	(10,25)	(2,5)	0.0224	0.0193	0.7787	0.5982	1.3018
	(15,25)	(3,5)	0.0209	0.0169	0.5815	0.4419	1.3155
	(25,25)	(5,5)	0.0183	0.0138	0.4741	0.3297	1.4381
0.850	(10,10)	(2,2)	0.0269	0.0175	0.9367	0.6756	1.3865
	(10,15)	(2,3)	0.0213	0.0145	0.6786	0.4552	1.4908
	(10,25)	(2,5)	0.0203	0.0126	0.5433	0.3479	1.5617
	(15,25)	(3,5)	0.0173	0.0108	0.4135	0.2598	1.5916
	(25,25)	(5,5)	0.0154	0.0083	0.3461	0.1962	1.7640

Table 2: MLE estimation of R when different values of the parameters α and β and when the scale parameter is known ($\lambda = 1$) and $r_x = r_y = 5$.

R	(n,m)	m_x,m_y	$Bias(\hat{R}_{s,\lambda})$	$Bias(\hat{R}_{r,\lambda})$	$MSE(\hat{R}_{s,\lambda})$	$MSE(\hat{R}_{r,\lambda})$	$eff(\hat{R}_{r,\lambda})$
0.250	(10,10)	(2,2)	0.0572	0.0404	1.3494	1.0606	1.2722
	(10,15)	(2,3)	0.0510	0.0367	0.9775	0.7147	1.3676
	(10,25)	(2,5)	0.0429	0.0320	0.7059	0.4944	1.4278
	(15,25)	(3,5)	0.0412	0.0299	0.5489	0.3729	1.4722
	(25,25)	(5,5)	0.0361	0.0230	0.4314	0.2397	1.8001
0.333	(10,10)	(2,2)	0.0445	0.0262	1.2644	1.0079	1.2545
	(10,15)	(2,3)	0.0368	0.0216	0.9609	0.6791	1.4149
	(10,25)	(2,5)	0.0300	0.0179	0.6570	0.4696	1.3990
	(15,25)	(3,5)	0.0291	0.0169	0.5101	0.3544	1.4391
	(25,25)	(5,5)	0.0229	0.0128	0.4020	0.2277	1.7651
0.500	(10,10)	(2,2)	0.0346	0.0226	1.2088	0.8948	1.3508
	(10,15)	(2,3)	0.0287	0.0191	0.8757	0.6028	1.4527
	(10,25)	(2,5)	0.0219	0.0151	0.6325	0.4169	1.5171
	(15,25)	(3,5)	0.0203	0.0137	0.4918	0.3146	1.5633
	(25,25)	(5,5)	0.0171	0.0113	0.3865	0.2022	1.9116
0.667	(10,10)	(2,2)	0.0320	0.0188	1.1450	0.8939	1.2808
	(10,15)	(2,3)	0.0247	0.0155	0.8044	0.5846	1.3760
	(10,25)	(2,5)	0.0164	0.0129	0.5711	0.4010	1.4243
	(15,25)	(3,5)	0.0156	0.0114	0.4351	0.2992	1.4543
	(25,25)	(5,5)	0.0129	0.0080	0.3341	0.1902	1.7563
0.850	(10,10)	(2,2)	0.0220	0.0130	0.7669	0.5005	1.5323
	(10,15)	(2,3)	0.0171	0.0107	0.5446	0.3372	1.6149
	(10,25)	(2,5)	0.0149	0.0084	0.3985	0.2332	1.7087
	(15,25)	(3,5)	0.0129	0.0073	0.3094	0.1759	1.7590
	(25,25)	(5,5)	0.0109	0.0048	0.2439	0.1132	2.1546

Table 3: MLE estimation of R when different values of the parameters α and β and when the scale parameter is known ($\lambda = 3$) and $r_x = r_y = 5$.

R	(n,m)	m_x,m_y	$Bias(\hat{R}_{s,\lambda})$	$Bias(\hat{R}_{r,\lambda})$	$MSE(\hat{R}_{s,\lambda})$	$MSE(\hat{R}_{r,\lambda})$	$eff(\hat{R}_{r,\lambda})$
0.250	(10,10)	(2,2)	0.4673	0.2989	3.0030	2.3605	1.2722
	(10,15)	(2,3)	0.3842	0.2457	2.0081	1.4393	1.3952
	(10,25)	(2,5)	0.3202	0.2138	1.4359	0.9957	1.4421
	(15,25)	(3,5)	0.2729	0.1788	0.9903	0.6726	1.4722
	(25,25)	(5,5)	0.2295	0.1460	0.7477	0.4591	1.6288
0.333	(10,10)	(2,2)	0.3630	0.1938	2.8139	2.2431	1.2545
	(10,15)	(2,3)	0.2595	0.1441	1.8447	1.3676	1.3489
	(10,25)	(2,5)	0.2239	0.1197	1.3364	0.9457	1.4131

	(15,25)	(3,5)	0.1927	0.1010	0.9202	0.6394	1.4391
	(25,25)	(5,5)	0.1457	0.0811	0.6967	0.4362	1.5971
0.500	(10,10)	(2,2)	0.2828	0.1670	2.6902	1.9915	1.3508
	(10,15)	(2,3)	0.2166	0.1278	1.7991	1.2139	1.4821
	(10,25)	(2,5)	0.1637	0.1008	1.2864	0.8395	1.5324
	(15,25)	(3,5)	0.1342	0.0823	0.8871	0.5675	1.5633
	(25,25)	(5,5)	0.1089	0.0719	0.6698	0.3873	1.7297
0.667	(10,10)	(2,2)	0.2614	0.1390	2.5482	1.9895	1.2808
	(10,15)	(2,3)	0.1863	0.1035	1.6525	1.1772	1.4038
	(10,25)	(2,5)	0.1226	0.0865	1.1617	0.8075	1.4386
	(15,25)	(3,5)	0.1035	0.0685	0.7849	0.5397	1.4543
	(25,25)	(5,5)	0.0820	0.0506	0.5791	0.3644	1.5892
0.850	(10,10)	(2,2)	0.1799	0.0958	1.7068	1.1139	1.5323
	(10,15)	(2,3)	0.1289	0.0718	1.1188	0.6791	1.6476
	(10,25)	(2,5)	0.1111	0.0565	0.8105	0.4696	1.7259
	(15,25)	(3,5)	0.0857	0.0438	0.5582	0.3173	1.7590
	(25,25)	(5,5)	0.0690	0.0305	0.4227	0.2168	1.9495

VI. CONCLUSIONS

In this paper, we have addressed the problem of estimating $R = P(Y < X)$ for the generalized inverted exponential distribution using both simple random sampling SRS and ranked set sampling RSS approaches. Comparing the performance of estimators of R, it is observed that:

1. The biases of all estimators is very small.
2. The estimators based on RSS dominate those based on SRS.
3. Relative to biases and MSEs, the ML estimator of R based on RSS is better than that based on SRS for different α, β and λ parameter values
4. Similarly, the Bayes estimator of R based on RSS is better than that based on SRS for different α, β and λ parameter values
5. Bayes estimator of R based on both approaches dominate the corresponding ML estimators.
6. The MSEs of estimators made using RSS is always less than those made by SRS.
7. Though out the simulation studies the estimation of R using the RSS approach is more efficient than that made using the SRS approach.
8. The efficiency of all estimators of based on RSS is decreasing as the set sizes is increasing.
9. This study revealed that the estimator of $P(Y < X)$ based on RSS technique is more efficient than the corresponding estimator based on SRS technique.

VII. FUTURE WORK

Many devices cannot function at high as well as low temperatures and a person's blood pressure has two limits (systolic and diastolic pressures). This case can be represented as the reliability parameter $R_3=P(X_1<Y<X_2)$, where Y is the stress that is limited by two strengths X_1, and X_2. In the near future estimation R_3 is considered for several lifetime distribution with real data application and using several sampling approaches.

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